



PANIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	LEVEL: 6
COURSE CODE: ODE 602S	COURSE NAME: ORDINARY DIFFERENTIAL EQUATIONS
SESSION: JANUARY 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR:	Dr I K O AJIBOLA

INSTRUCTIONS
<ol style="list-style-type: none">1. Answer ALL the questions in the booklet provided.2. Show clearly all the steps used in the calculations.3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [20marks]

1. (a) Given that

$$\frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)}$$

is homogeneous, then obtain the separation of variable.

(10)

- (b) Solve

$$x^2 y'(x) = 1 - x^2 + y^2 - x^2 y^2, \quad y(1) = 0.$$

(10)

QUESTION 2 [20 marks]

2. (a) Given a differential equation

$$y'(x) = a(x)y^2 + b(x)y + c(x)$$

and one particular solution $y_1(x)$ is known, show that $y(x) = y_1(x) + z(x)$ can transform this equation into Bernoulli equation.

(10)

- (b) Find the general solution to the following differential equation

$$y'(x) - 13(x^2 + y^2) + 26xy = 1, \quad y_1(x) = x$$

(10)

QUESTION 3 [20 marks]

3. (a) If
- $y_1(x)$
- and
- $y_2(x)$
- are solutions of
- $y'' + p(x)y' + q(x)y = 0$
- , show that the general solution of

$$y'' + p(x)y' + q(x)y = f(x)$$

is

$$y_G(x) = c_1 y_1(x) + c_2 y_2(x) + \left(- \int \frac{y_2(x)f(x)}{W} dx \right) y_1(x) + \left(\int \frac{y_1(x)f(x)}{W} \right) y_2(x)$$

(10)

- (b) Solve the Euler equation

$$x^2 y''(x) - 5x y'(x) + 10y(x) = 0, \quad y(1) = 4, \quad y'(1) = -6$$

(10)

QUESTION 4 [20 marks]

4. Given the ordinary differential equation

$$(1 - x^2)y''(x) - 2xy'(x) + 6y(x) = 0$$

- (a) Find two independent series solution $y_1(x)$ and $y_2(x)$ (12)
- (b) Determine the radius of convergence and interval of convergence of given differential equation in question 4. (8)

QUESTION 5 [20 marks]

5. (a) A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at the same rate.
- Find the number $A(t)$ of pounds of salt in the tank at time t . (2)
 - what is the concentration $c(t)$ of the salt in the tank at time t and at time $t = 5$? (2)
 - What is the concentration of the salt in the tank after a long time? (2)
 - At what time is the concentration of the salt in the tank equal to one-half this limiting value? (2)
 - Solve this Problem under the assumption that the solution is pumped out at a faster rate of 10 gal/min. When is the tank empty? (2)
- (b) Find the function $y_2(x)$, if

$$y_1(x) = e^{-2x}, \quad \text{the Wronskian } W(y_1, y_2) = (2x - 3)e^{-2x} \quad \text{and} \quad y_2(0) = -2.$$

(10)

End of Exam!